

Forecasting Volatility of Currencies and Oil with Brown Firms: A Mixed-Frequency Approach

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Abstract

Climate risk is unequal: it affects different countries and assets differently, with important impacts on currencies and energy markets. And financial markets embed forward-looking information about climate risk expectations (Schlenker and Taylor (2021); Downey et al. (2023); Hale (2024)). We examine whether firms' climate exposure helps forecast realized volatility of commodity currencies and oil futures. To capture information about climate risk that arises at multiple frequencies, we construct 'green' and 'brown' indices using a mixed data frequency state-space model. Our dataset combines intraday prices for commodity currencies (AUD, EUR, CAD, and NOK), oil futures (WTI and Brent), and high frequency equity data for green and brown firms. We use a heterogeneous autoregressive (HAR) framework and assess robustness with a Long Short-Term Memory (LSTM) model. The *brown* index significantly enhances out-of-sample forecasts for currency volatility, outperforming green and benchmark models. Our brown index aggregates signals from several economic channels. In particular, beyond the influence of global risk premia and policy uncertainty, it captures broad risks encompassing commodity prices and energy supply.

Keywords: Brown Firms; Currency; Energy Market; Economic Channels; Volatility Forecast; Machine Learning; Mixed-Frequency

JEL Classification: C22; C53; F31; F64; Q02; Q47; Q54

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1 Introduction

Climate risk affects global financial markets through multiple channels, influencing both exchange rates and energy prices. Exchange rates embed information about global disaster and macroeconomic risks (Farhi and Gabaix (2016)), while oil and other energy markets are directly impacted by climate policy, transition uncertainty, and carbon price shocks (Rao et al. (2023)). Despite these linkages, the two markets are seldom studied jointly, and little is known about how climate risk exposure of firms contributes to volatility dynamics across both. Understanding these cross-market effects is crucial, as volatility in exchange rates shapes international portfolio allocation and policy responses, while volatility in energy markets affects inflation, national income, and geopolitical stability. Moreover, environmental policies can produce unexpected spillovers—so-called “leakage” effects—across sectors and markets (Fowlie and Reguant (2022); Hernández-Cortés (2025)). Investors and policymakers may therefore overlook valuable information if they ignore how climate-related risks influence volatility in currency and energy markets.

This paper fills that gap by developing new measures of firms’ climate risk exposure and linking them to forecasting performance in foreign exchange and oil futures markets. Specifically, we distinguish between “green” firms—those with low environmental risk—and “brown” firms—those with high carbon intensity and exposure to transition and physical climate risks. We then use these classifications to extract latent volatility factors that reflect climate-related financial stress. Our analysis is motivated by the idea that markets incorporate climate information continuously and at multiple temporal frequencies, ranging from high-frequency trading to longer-term policy expectations. Capturing this structure requires methods that can synthesize information across different time horizons.

Our first contribution is to construct novel green and brown volatility indices that measure firms’ exposure to climate-related risk. We use firm-level data from MSCI ESG Ratings, focusing on the Environmental pillar score (`E_score`) and Environmental pillar weight (`E_weight`) to classify firms according to their greenness or brownness. Based on market capitalization, we select the largest green and brown firms and construct weighted portfolios for each group. Using high-frequency stock prices from the WRDS TAQ database, we compute realized volatilities for these portfolios at daily, weekly, bi-weekly, and monthly frequencies. This design enables us to capture how climate risk manifests across different time scales, from intraday financial shocks to longer-term market adjustments.

Our second contribution is methodological. We employ a mixed-frequency state-space model (Peng et al. (2024)) to extract latent volatility factors that integrate information across these multiple horizons. The resulting mixed-frequency green and brown indices summarize high- and low-frequency volatility information into unified measures of climate exposure. This approach bridges the gap be-

tween firm-level microstructure and macro-level market volatility, providing a new way to quantify how climate risks are priced into financial markets. To our knowledge, this is the first application of a mixed-frequency approach to construct climate-based volatility factors and to link them simultaneously to currency and energy markets.

Our third contribution is empirical. We use the green and brown indices to forecast realized volatility in both commodity currency exchange rates (AUD, EUR, CAD, and NOK) and oil futures (WTI and Brent). We begin with the heterogeneous autoregressive (HAR) model of realized volatility (Corsi (2009)) as our benchmark, then augment it with the green and brown indices. To assess robustness, we also employ a machine learning approach based on a Long Short-Term Memory (LSTM) neural network (Hochreiter and Schmidhuber (1997)). Both models are estimated using high-frequency data from January 2015 to November 2023. Our results reveal a consistent and economically meaningful pattern: the *brown volatility index* significantly enhances out-of-sample forecast accuracy, particularly for foreign exchange markets. Models augmented with the brown index outperform those with only the green index, as well as benchmark models using the oil volatility index (OVX). The predictive improvement is especially pronounced during high-volatility regimes, indicating that brown-sector volatility transmits early warning signals about macro-financial stress.

Our fourth contribution is to examine the economic channels through which the brown index captures information relevant to volatility in currency and oil markets. Rather than evaluating out-of-sample forecasting performance, we conduct in-sample regressions to assess the statistical significance of these mechanisms. Specifically, we test three channels that link brown-firm volatility to broader market fluctuations. First, the commodity price and energy supply channel reflects how brown firms—dominated by oil, gas, and resource companies—transmit shocks associated with energy supply constraints and commodity price dynamics. We find that the sensitivity of realized volatility to the brown index increases when oil inventories are low, consistent with volatility transmission through energy-market tightness. Second, the industrial transition and policy uncertainty channel captures how policy ambiguity and transition risk affect brown-sector firms. Incorporating measures of climate policy uncertainty shows that, while policy uncertainty itself adds limited explanatory power, the brown index remains a statistically significant predictor, suggesting it endogenously embeds information about transition-related risks. Third, the global risk premium and hedging channel links brown-sector volatility to broader financial stress via derivatives hedging and liquidity adjustments. Controlling for the VIX, the brown index continues to exhibit significant in-sample explanatory power, indicating that it captures risk-transmission mechanisms not fully reflected in standard global risk indicators.

Taken together, our findings provide strong evidence that brown firms' volatility embodies a composite signal of climate–energy–financial risk. It aggregates information about commodity market stress,

transition uncertainty, and global risk premia into a single measure that improves forecasting performance for both exchange rate and oil market volatility. The results suggest that carbon-intensive firms play an outsized role in transmitting climate-related shocks across international markets. For investors, the brown index offers a forward-looking indicator of systemic risk under climate and energy transitions. For policymakers, it highlights the importance of monitoring climate-sensitive sectors as part of financial stability analysis.

Overall, this paper contributes to the growing literature on climate finance and market volatility by integrating high-frequency financial data, mixed-frequency econometric modeling, and climate risk measurement into a unified framework. By showing that brown-sector volatility carries predictive information for key global assets, we provide new evidence on how climate risk propagates across markets, and offer a practical tool for forecasting and risk management in an increasingly climate-sensitive global economy.

The remainder of the paper is organized as follows. Section 2 reviews the related literature on climate risk, market volatility, and mixed-frequency modeling. Section 3 describes the data and methodology. Section 4 presents the main empirical findings on volatility forecasting performance using both HAR and LSTM frameworks. Section 5 examines the economic channels and mechanisms through which the brown index captures information about volatility dynamics in currency and oil markets. Section 6 concludes and discusses the broader implications of our findings for investors, policymakers, and future research on climate-related financial risk.

2 Literature Review

We build on a vibrant young literature examining climate risk’s link to two main areas: energy, derivatives, and adjustment costs; and international markets.¹ In the first area, energy and derivatives market research has paid a lot of attention to forecasting volatility with sophisticated time series models, some of which incorporate climate concerns. Indeed, the price of oil sometimes suffers steep declines or large upswings. Such swings are sometimes predictable, and attributable to a combination of demand shocks and current or expected production shocks (Baumeister and Kilian (2016)). Research by Munasinghe et al. (2012) studies the implications of climate change for record high temperatures. The authors develop a metric for record temperatures, which they use to assess climate change globally. They document a tenfold increase in the frequency of extreme temperatures between the first three decades of the twentieth century and the first decade of the twenty-first century. They uncover larger increases

¹For broader surveys of sustainable finance and related volatility literature, see Gillan et al. (2021); Edmans and Kacperczyk (2022); Starks (2023); and Cholleto et al. (2024).

in the tropics, and find an increased frequency of both record lows and record highs. Ma et al. (2017) extend the workhorse heterogeneous autoregressive (HAR) model of realized volatility, by accounting for regime switches. The authors apply their framework to forecast realized volatility in the crude oil futures market. They document existence of a transient high volatility regime, and that their framework outperforms the basic HAR model out of sample. Demirer et al. (2018) study predictive power of rare disaster risk for oil market returns and volatility using a nonparametric approach. The authors detect a nonlinear relationship and structural break between oil returns and rare disaster risks. Moreover, they find that rare disasters strongly affect oil returns and volatility.

Related work by Guidolin and Pedio (2021) assesses whether three commodity-based variables have additional predictive power for commodity futures, beyond prediction by macroeconomic factors. The authors compute out of sample forecasts for fifteen commodity futures, using a weighted regime switching model for volatility. These forecasts are compared to an AR(1) model's forecasts. The authors document that commodity-specific factors do not improve forecasting power, although they may help investment strategies during high-volatility periods. Schlenker and Taylor (2021) examine whether derivatives reflect climate expectations. The researchers compare prices of weather-based financial derivatives to climate model predictions and weather station data from eight cities in the US. They discover that derivative prices respond both to short-term weather forecasts and long-term trends. Moreover, the authors conclude that investors are focusing on scientific projections, because futures prices reflect climate model predictions rather than actual weather station trends. Downey et al. (2023) study the effect of rainfall volatility on the construction industry. They document a decrease in employment with forecasted rainfall. The authors then estimate the impact on adjustment costs. They show that firms pay 10% of their monthly profit to adjust when rainfall is anticipatable one month in advance, while paying less than 1% when rainfall is anticipatable six months in advance. They conclude that other things equal, increased rainfall volatility due to climate change will increase firms' adjustment costs. Rao et al. (2023) analyse spillovers from climate risk and carbon emissions uncertainty to energy markets. The authors consider monthly and daily frequencies of climate policy uncertainty (CPU) and carbon emissions (CO). The connectedness for CO and energy prices is most pronounced at medium horizons (3–6 months), while the connectedness for CPU is most pronounced for longer horizons of 6 months or more. They document that CPU is a net transmitter to energy prices in horizons below 3 months. By contrast, CO is a net transmitter to natural gas for 1 month and 1–3 months, and a net transmitter for oil over the 3–6 month horizon. We add to this exciting research by studying the volatility of both currencies and energy with modern forecasting and machine learning techniques.

In the second area, international implications of climate risk have recently been studied in both trade and financial settings. Gu and Hale (2023) examine whether multinational firms react to climate-

related risks by altering their foreign direct investment (FDI) in affected countries. The authors develop a theoretical model where firms consider climate risks when deciding where to locate. According to their model, FDI should decrease when climate risks increase, but FDI has an ambiguous interaction with emission productivity. They document some empirical support for the model, but no consistent evidence for climate risks affecting FDI at the country level. Firm level data reveals that in the aftermath of the Paris Climate Accord, firms with large exposure to climate risk reduce FDI more. Bonato et al. (2023) devise a comprehensive study using climate risks to forecast realized volatility of exchange rates in key fossil fuel exporters. The authors capture climate risk with a variety of metrics, and forecast with random forests. They show that the incorporation of climate risks can improve forecast performance for realized volatility of exchange rates, and in a simulation demonstrate the potential for economic gains. Hale (2024) assesses the role of belief formation in determining how climate disasters affect real exchange rates. The researcher extends the Farhi and Gabaix (2016) to account for belief formation, and simulates the model for 47 countries. She documents that differences in belief formation have scant effect on model fit, and that Bayesian beliefs do not perform better than others. We add to this new literature by forecasting realized volatility of both currencies and oil, and by utilizing a sophisticated latent-variable method that aggregates climate risk information that arises at multiple frequencies.

Beyond the above literature, we build on recent papers in multi-frequency analysis and forecasting with climate-based portfolios. Peng et al. (2024) develop a state space methodology to extract multi-frequency factors from financial and macroeconomic data. The state space model includes multiple frequencies of quantities from various sources, including estimated components of quadratic variation and macroeconomic data. The researchers use their framework to forecast growth rates of monthly economic variables as well as corporate bond yields, and document improved forecasting performance for both cases. Related work by Yu et al. (2024) constructs a mixed-frequency uncertainty factor, which they use to forecast the realized volatility of Shanghai oil futures. The uncertainty factor incorporates multiple frequency information on macroeconomic variables such as geopolitical risk, market uncertainty, and high-frequency financial observations. The researchers document that their uncertainty factor possesses excellent forecasting performance for volatility, both in- and out of sample. Chollete et al. (2024) form daily portfolios of green and brown firms by extending the methodology of Pástor et al. (2021). The authors analyze realized volatility for green and brown firms, using GARCH, machine learning, and historical volatility. They report similarity between unconditional volatilities of brown and green firms. By contrast, they discover broad difference in the forecasting power for volatility. In particular, it is much harder to forecast volatility for green firms than for brown firms. We contribute to this literature by extending the portfolio formation in Chollete et al. (2024) to intraday and weekly settings, and at the same time providing a new application of the mixed frequency model of Peng et al. (2024) to forecast

currency and oil market volatility.

3 Data and Methodology

We collect data on four commodity currency exchange rates (AUD, EUR, CAD, and NOK) together with WTI and Brent oil prices from `HistData.com`. The dataset consists of 1-minute trading intervals spanning the period from January 5, 2015, to November 30, 2023. We resample the data into 5-minute frequency. In addition, we construct green and brown portfolios using MSCI’s ESG Ratings dataset. Our approach follows that of Chollete et al. (2024). Specifically, we employ the variables *Environmental pillar score* (`E_score`) and *Environmental pillar weight* (`E_weight`) to quantify each firm’s degree of greenness, in line with the methodology described in Pástor et al. (2022).

We match firms in the MSCI dataset with common stocks (share codes of 10 or 11) in the CRSP monthly dataset. We first match based on the first 8-digit CUSIP code. If a firm does not have a match using the 8-digit CUSIP, we then try matching it using the 6-digit CUSIP. In this case, we only include the firm if it is the only class of stock issue available and there are regular updates to its ESG rating. If a firm does not have a matching CUSIP but has the same TICKER on both databases, we include it only if it has the same company name, operates in the same industry, and regularly updates its ESG scores.

We proceed in two steps. First, we construct weighted portfolios for both green and brown firms by selecting the 11 largest green firms and the 12 largest brown firms, based on market capitalization. Utilizing high-frequency stock prices sampled at 5-minute intervals, we compile data spanning from January 2, 2015, to November 30, 2023. The high-frequency trading data are obtained from the WRDS TAQ database. We then compute the realized volatility of the green and brown portfolios at daily, weekly, bi-weekly, and monthly frequencies using the 5-minute data. Second, we apply a Kalman filter within a state-space framework to extract latent indices for green and brown firms, based on the realized volatilities computed at the different time intervals.

Table 1 presents a breakdown of the Green and Brown portfolios’ composition. The Green portfolio is split evenly between health care, financials, and communication services, while the Brown portfolio is dominated by firms in the oil and gas complex— *Chevron (CVX)*, *Valero (VLO)*, *Phillips 66 (PSX)*, *Marathon Petroleum (MPC)*, *Kinder Morgan (KMI)*, and *Williams Companies (WMB)*. Thus, the Brown Index directly reflects fluctuations in global energy markets. Table 2 presents summary statistics on the currencies, oil prices, and Green and Brown indices. While the mean and standard deviations are relatively similar for the Green and Brown indices, they differ in shape: the Brown index is much more skewed and heavy tailed.

3.1 Realized Volatility Measure

As outlined by Andersen et al. (2007), the concept of realized volatility (RV) is a widely adopted approach for measuring an asset's return variability within a given trading day. Realized volatility is calculated by summing the squared returns observed over high-frequency intraday intervals. Mathematically, it is expressed as follows:

$$RV_t = \sum_{i=1}^n r_{i,t}^2, \quad (1)$$

where $r_{i,t} = X_{i\Delta_n,t} - X_{(i-1)\Delta_n,t}$ represents the intraday log return observed over the i -th interval, and X_t denotes the log price of the asset. The term Δ_n indicates the sampling interval, and n refers to the total number of intraday observations for that particular trading day.

3.2 'Green' and 'Brown' Uncertainty Indices

Following the methodology developed by Aruoba et al. (2009) and Peng et al. (2024), we implement a standard state space model to extract uncertainty factors at multiple time intervals. Our observation equation incorporates a dependent variable defined as $y_t = [RV_t^d, RV_t^w, RV_t^{bi-w}, RV_t^m]'$, which includes observations recorded of realized volatility of daily (d), weekly (w), bi-weekly ($bi-w$) and monthly (m). By employing this mixed-frequency approach, we construct mixed-frequency factors. The unobserved latent factor generated from the state space model is denoted by F_t . The state space model framework is described as follows:

Observation Equation:

$$y_t = CF_t + \eta_t, \quad (2)$$

State Equation:

$$F_{t+1} = \phi F_t + D\epsilon_t,$$

where the error term follows a normal distribution with mean zero and variance Q . This mixed-frequency state space model extends the conventional approach by incorporating accommodations for missing observations in lower-frequency variables.

In the single-frequency case, the data structure simplifies to $y_t^* = y_t$. For the mixed-frequency setting, we define a matrix M_t , where elements are assigned as 0 for missing values and 1 for observed values in the signal vector. Consequently, the mixed-frequency data structure is represented as:

$$y_t^* = M_t y_t. \quad (3)$$

To align with this formulation, the latent state vector F_t is expressed as a $1 \times q$ vector of state variables. For each time period t (where $t = 1, \dots, T$), the mixed-frequency state space model is specified as:

$$y_t^* = G^* F_t + M_t \eta_t, \quad (4)$$

$$F_{t+1} = \phi F_t + D \epsilon_t. \quad (5)$$

Since y_t^* is a linear transformation of y_t , we can still estimate the state vectors by integrating this transformation into the standard state space framework. Specifically,

$$y_t^* = M_t y_t, \quad (6)$$

$$G^* = M_t C. \quad (7)$$

Building on the framework of Aruoba et al. (2009), we specify the joint distribution of the mixed-frequency state space model as:

$$\begin{pmatrix} F_t \\ y_t^* - E(y_t^* | y_{t-1}^*) \end{pmatrix} \sim N \left[\begin{pmatrix} F_{t|t-1} \\ 0 \end{pmatrix}, \begin{pmatrix} P_t G^{*'} & P_t G^{*'} \\ P_t G^{*'} & S_t \end{pmatrix} \right], \quad (8)$$

where $P_t G^{*'}$ is the variance of F_t conditional on y_{t-1}^* , and S_t is the variance of $y_t^* - E(y_t^* | y_{t-1}^*)$. Using this distribution, the updated state vector and its associated variance are computed as follows:

$$F_{t|t} = F_t + P_t G^{*'} S_t^{-1} [y_t^* - E(y_t^* | y_{t-1}^*)], \quad (9)$$

$$P_{t|t} = P_t + P_t G^{*'} S_t^{-1} G^* P_t'. \quad (10)$$

The updated state variables are iteratively calculated as:

$$F_{t+1} = \phi F_{t|t}, \quad (11)$$

$$P_{t+1} = \phi P_{t|t} \phi' + D Q D'. \quad (12)$$

3.3 Forecasting Models

We now assess whether the green and brown firm volatility indices can enhance the forecasting accuracy of realized volatility for key commodity currency exchange rates and oil futures, specifically the Australian dollar (AUD), the Euro (EUR), the Canadian dollar (CAD), the Norwegian kroner (NOK), WTI, and Brent futures. The realized volatilities of these exchange rates and oil futures are also derived from 5-minute high-frequency data. We use two main forecasting models, heterogeneous autoregressive and Long short-term memory approaches.

Our first, benchmark forecasting model is the heterogeneous autoregressive realized volatility (HAR-RV) framework based on Corsi (2009):

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \epsilon_{t+h}, \quad (13)$$

where RV_{t+h} is the h -period ahead forecast; RV_t denotes daily realized volatility at period t , $RV_{w,t}$ denotes weekly realized volatility at period t ; $RV_{m,t}$ denotes the monthly realized volatility at period t ; and ϵ_{t+h} is the error term. We extend this model by incorporating the green and brown firm volatility indices I_{Green} and I_{Brown} to evaluate their impact on out-of-sample forecasting performance. We therefore estimate the following augmented models:

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \delta_d I_{Green,t} + \epsilon_{t+h} \quad (14)$$

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \delta_d I_{Brown,t} + \epsilon_{t+h} \quad (15)$$

$$RV_{t+h} = \beta_0 + \beta_d RV_t + \beta_w RV_{w,t} + \beta_m RV_{m,t} + \delta_d I_{Green,t} + \gamma_d I_{Brown,t} + \epsilon_{t+h} \quad (16)$$

Intuitively, we add brown and green indices—separately then in concert—to assess whether they improve the initial benchmark model in (13). We utilize a rolling window of 600 days to forecast 1-day-ahead realized volatility. The results indicate that the brown firm volatility index significantly improves out-of-sample forecasting performance. Although the green firm volatility index also enhances forecasting accuracy, its impact is less pronounced compared to the brown firm volatility index. We evaluate out-of-sample performance using metrics such as mean square forecasting error (MSFE) and

out-of-sample R^2 . In the next phase, we shall incorporate the LSTM machine learning model, to further assess forecasting performance of the green and brown firm volatility indices.

Our second model is the Long Short-Term Memory (LSTM) model, introduced by Hochreiter and Schmidhuber (1997), a special recurrent neural network (RNN) designed to capture long-term dependencies in sequential data. The architecture of an LSTM network incorporates memory cells and gating mechanisms, which mitigate the vanishing gradient problem that traditional RNNs often encounter.

Consider a sequential dataset with input vectors denoted as $x_t \in \mathbb{R}^d$ at each time step t . The LSTM model maintains a hidden state vector $h_t \in \mathbb{R}^h$ and a cell state vector $c_t \in \mathbb{R}^h$ to store long-term information. The cell state is updated using three gating mechanisms: the input gate, forget gate, and output gate.

The input gate, denoted as i_t , controls the extent to which new information is added to the cell state. The forget gate, f_t , determines the proportion of the previous cell state to retain. The output gate, o_t , governs how much of the cell state is output to the next layer or prediction step. These gates are defined as follows:

$$i_t = \sigma(W_i x_t + U_i h_{t-1} + b_i) \quad f_t = \sigma(W_f x_t + U_f h_{t-1} + b_f) \quad o_t = \sigma(W_o x_t + U_o h_{t-1} + b_o) \quad (17)$$

Here, $\sigma(\cdot)$ represents the sigmoid activation function, and W , U , and b denote weight matrices and bias terms, respectively. The cell state is updated according to:

$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t \quad (18)$$

where $\tilde{c}_t = \tanh(W_c x_t + U_c h_{t-1} + b_c)$ represents the candidate cell state, and $\odot$ denotes element-wise multiplication. The updated hidden state is computed as follows:

$$h_t = o_t \odot \tanh(c_t) \quad (19)$$

By incorporating these mechanisms, the LSTM model effectively maintains information over extended time periods while mitigating the vanishing gradient problem. Consequently, LSTMs have been widely adopted in various applications such as time series forecasting, natural language processing, and financial modeling.

3.4 Evaluation Metrics

To implement the HAR-type forecasting models, we adopt a dynamic rolling window approach with a window size of 600 trading days. This method allows for adaptive model updating to account for evolving data patterns. The sample period is divided into two distinct phases: the in-sample estimation period (January 5, 2015 - June 21, 2017) and the out-of-sample forecasting period (June 22, 2017 - November 30, 2023). For each forecasting step, the model is re-estimated using the most recent 600-day window of data, ensuring that the forecast for the subsequent period is based on the most up-to-date information available.

For the Long Short-Term Memory (LSTM) network, we partition the sample data into three distinct subsets: training, validation, and test datasets. The data is allocated following a 30-20-50 split, where 30% of the data is reserved for training, 20% for validation, and the remaining 50% for testing.

We evaluate the predictive accuracy of our out-of-sample forecasting results using two key performance metrics: the root mean square error (RMSE) and the out-of-sample R^2 statistic. Moreover, we assess the statistical significance of forecasting performance differences across models, using the Model Confidence Set (MCS) methodology of Hansen et al. (2011). The MCS procedure iteratively excludes the weakest-performing models from an initial set $\mathcal{M}$ until the null hypothesis of equal predictive accuracy (EPA) can no longer be rejected at a designated significance level. The resulting set of superior models is defined as the MCS, denoted $\mathcal{M}_{1-\alpha}^*$, which comprises models that achieve the highest performance within a $1 - \alpha$ confidence level.

To test the EPA null hypothesis, a test statistic is constructed as follows:

$$t_{i,j} = \frac{\bar{d}_{i,j}}{\sqrt{\widehat{\text{var}}(\bar{d}_{i,j})}}, \quad i, j \in \mathcal{M}$$

where

$$\bar{d}_{i,j} = \frac{1}{n} \sum_{t=1}^n d_{i,j,t}$$

measures the relative loss between models i and j , with $d_{i,j,t} = L_{i,t} - L_{j,t}$. The null hypothesis asserts that $H_{i,j} : E(d_{i,j,t}) = 0$. Consequently, the null hypothesis is evaluated using the following test statistics:

$$T_R = \max_{i,j \in \mathcal{M}} |t_{i,j}|$$

$$T_{SQ} = \sum_{i,j \in \mathcal{M}} t_{i,j}^2$$

Within the initial set, p-values are computed for each model. A model is included in the final MCS

set if its corresponding p-value, denoted $\hat{p}_i$, satisfies the condition $\hat{p}_i \geq \alpha$. This ensures that only the most robust models are retained for subsequent analysis. The p-value represents the likelihood that a model is not significantly worse than the best model in the set. A p-value of 1 indicates that the model is never rejected during testing and consequently, is very likely to be in the set of best models. Conversely, a p-value of 0 indicates that the model is always rejected as significantly worse than others, and should be excluded from the set of best models.

3.5 Markov Regime Switching Model

While we focus on the abovementioned LSTM model for our forecasting exercises, we also incorporate separate robustness checks with regime switching models. These models build on the statistical literature on hidden markov processes, to capture economic variables that exhibit nonstationary shifts or regimes over time (Hamilton (1989)). Each latent regime has its own parameters, and a markov model characterizes the probability of transitioning between regimes. Regime switching models have been fruitfully applied in finance, characterizing abrupt changes in volatility or risk, among other key financial quantities (Ang and Bekaert (2002); Guidolin and Timmermann (2008)).

In our context, we represent a regime switching model for the time series y_t of currency returns in the following manner. We allow y_t 's mean μ and variance σ^2 to switch, depending on an unobserved variable S_t :

$$y_t = \mu_{S_t} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_{S_t}^2).$$

S_t is a state variable that changes depending on whether the market is in a "bull" or "bear" regime: $S_t = 1$ in a bull market, and 2 in a bear market. The parameters μ_{S_t} and $\sigma_{S_t}^2$ change depending on what the current regime is. The transition from one regime to another depends on a probability matrix P . The elements of P are p_{ij} for $i, j = 1, 2$, which represent the probability of moving from regime i to regime j .

4 Results

We present our mixed-frequency factors in Figures 1 and 2. Figure 1 presents the brown volatility factor in red, overlaying the different frequencies. Evidently the different frequencies coincide a great deal for moderate observations, while there is some heterogeneity in the size of jumps. The series are dominated by an extreme upward jump in early 2020, which may reflect market uncertainty at the start of the *Covid-19* pandemic. Figure 2 shows a similar pattern for the green factor.

In Figure 3 we present rolling correlations between the green and brown factors. The two factors

tend to have relatively large correlations, although they dropped to zero and negative values in 2021. The correlations rose again at the end of 2022, but since mid-2023 have once again fallen. Such comovement of the factors may reflect international factors at play such as (expectations of) climate regulation and geopolitical conflict.

We present our initial forecast results in Table 3. Our benchmark model for each currency and commodity is the HAR model, which we add a green factor, a brown factor, or both. Our chief finding is that the addition of the Brown factor alone to HAR always reduces MSE more than when we add the green factor. Indeed, with the exception of the WTI oil market, adding the brown factor alone reduces forecast error more than adding both green and brown factors. Moreover, the out of sample R^2 is always highest for HAR plus the brown factor—even higher than HAR plus both green and brown factors. This finding holds for all currencies and for both oil markets.

A natural question concerns the generality of our results. With this question in mind, we proceed in two steps. First, we explore correlations for multiple frequencies. Second, we expand our model from Table 3 to account for the Oil Volatility Index OVX, and explore a further forecasting analysis with LSTM machine learning framework. Figure 4 presents the rolling correlations with windows spanning from 30 days to 252 days. While the correlation’s lower bound naturally increases with window length, the general pattern remains: the correlations are overwhelmingly positive and feature a steep decline in 2021. In Table 4 we add the oil volatility index OVX to our comparison. This addition makes sense because of the volatility of oil prices is already an important consideration for investors. It is therefore important to understand whether our indices convey additional forecasting benefits beyond OVX. For the currencies MSE is typically larger for the model with OVX and out of sample R^2 is lower, indicating that OVX does worse than the brown index for forecasting currency volatility.

We turn to a machine learning model of the Long Short Term Memory (LSTM) format in Table 5. Machine learning techniques are useful in this context because they provide an alternative perspective on the HAR approach to forecasting, and allow us to assess robustness of our results. Here, our benchmark model is LSTM, to which we then add green, brown, and OVX factors. The results align reasonably well to those in Table 4: in particular, the OVX is more predictive than green and brown factors for oil futures, However, OVX once again does worse than either brown or green indices for forecasting currency volatility. Thus, the picture that emerges from the preceding analysis is that a mixed-frequency approach to constructing indices can meaningfully improve forecasting of currencies. The improvement is often more notable for an index based on brown firms.

To further evaluate the statistical significance of the relative out-of-sample forecasting performance, we employ the Model Confidence Set (MCS) test. The results, reported in Table 5 and Table 6, include assessments for benchmark HAR models, LSTM models, and four alternative models that incorporate

various predictive factors: the green factor, the brown factor, a combination of green and brown factors, and the OVX index. For one-day-ahead forecasts, the benchmark HAR model augmented with the brown factor achieved an MCS p-value of 1 when predicting foreign exchange rates for the AUD, EUR, CAD, and NOK. This result indicates that the brown factor-enhanced model demonstrated superior predictive performance relative to its counterparts. These findings provide robust evidence that incorporating the brown factor improves forecasting accuracy in foreign exchange markets. Conversely, when forecasting oil futures (including WTI and Brent), the benchmark HAR model augmented with the OVX index attained an MCS p-value of 1. Thus, the OVX-enhanced model outperforms competing models, indicating that the OVX index possesses greater predictive power than the green and brown factors in the context of oil futures forecasting.

We further evaluate the forecasting performance of HAR-type and LSTM-type models under different volatility regimes. Specifically, we use the VIX as a proxy for market volatility and classify the forecasting periods into high- and low-volatility regimes. The results, presented in Table 7 and Table 8, compare the predictive accuracy of these models across the two regimes. A key finding is that the predictive power of all models tends to be higher in high-volatility regimes and lower in low-volatility regimes. Moreover, the brown factor appears to play a more significant role during periods of elevated market stress, exhibiting stronger predictive power in high-volatility conditions, particularly in forecasting foreign exchange rates include AUD, EUR, CAD, and NOK. This finding suggests that brown firms, perhaps as a result of their heightened exposure to climate risks and fluctuations in energy markets, may serve as indicators of market volatility when systemic stress intensifies.

5 Economic Channels and Mechanisms

Our foregoing results establish strong evidence on the statistical importance of the Brown factor. We now delve into economic channels that drive this importance. We focus on three channels: commodity price and energy supply, industrial transition and policy uncertainty, and global risk premia.

5.1 Commodity Price and Energy Supply Channel

A key mechanism through which the Brown Index may forecast exchange rate volatility operates via the commodity price and energy supply channel. It has long been recognized that commodities prices have subtle but strong implications for currency movements (Bui and Pippenger (1990); Wang and Cheung (2023)). The Brown portfolio is dominated by firms in the oil and gas complex, including *Chevron (CVX)*, *Valero (VLO)*, *Phillips 66 (PSX)*, *Marathon Petroleum (MPC)*, *Kinder Morgan (KMI)*, and *Williams Com-*

panies (*WMB*). Given this composition, the Brown Index directly reflects fluctuations in global energy markets and, especially, in the volatility of oil and gas prices. These firms are exposed to fundamental energy supply shocks—such as OPEC production decisions, geopolitical conflicts, and changes in global inventories—which propagate rapidly into their equity valuations and volatility. Periods of tight inventories, when oil stocks are low relative to demand, are especially important: small supply shocks in such periods tend to generate disproportionate price responses, elevating the volatility of energy-sector equities and, by extension, the Brown Index itself.

Consequently, the Brown Index can be interpreted as a latent measure of energy market volatility that transmits into other financial markets. The transmission operates through two principal mechanisms. First, changes in oil and gas prices alter the terms of trade of commodity-exporting economies, such as Australia, Canada, and Norway, thereby affecting the volatility of their exchange rates. Second, volatility in energy prices influences the risk premia embedded in oil futures markets, as speculative and hedging activity respond to perceived supply constraints. Through these channels, the Brown Index aggregates information about both financial and physical conditions in global energy markets, linking the volatility of carbon-intensive firms to the broader dynamics of commodity and currency markets.

5.1.1 Empirical Specification and Findings

To test for this mechanism, we extend the standard Heterogeneous Autoregressive (HAR) model of realized volatility Corsi (2009) by incorporating measures of brown-sector volatility and energy supply tightness. The model is given by:

$$RV_{t+1} = \alpha + \beta_d RV_t^d + \beta_w RV_t^w + \beta_m RV_t^m + \delta Brown_t + \gamma Inv_t + \eta(Brown_t \times Inv_t) + \varepsilon_{t+1}, \quad (20)$$

where RV_{t+1} denotes the next-day realized volatility of currency returns; RV_t^d , RV_t^w , and RV_t^m represent the daily, weekly, and monthly realized volatility components, respectively; $Brown_t$ is the Brown firm volatility index; and Inv_t denotes a standardized measure of U.S. oil inventories.² The interaction term $Brown_t \times Inv_t$ captures how the predictive power of the Brown Index varies with energy supply conditions. If the forecasting strength of the Brown Index intensifies during periods of low oil inventories, we expect $\eta < 0$. Such a finding would suggest that the Brown Index transmits volatility more effectively when the oil market is tight and therefore more susceptible to supply shocks.

Table 15 reports estimation results using daily data over the period 2015–2023. The Brown Index remains statistically significant in all specifications, confirming its importance as a predictor of realized

²We use the U.S. Energy Information Administration (EIA) Weekly U.S. Ending Stocks of Crude Oil (Thousand Barrels).

volatility. When oil inventories are added to the baseline HAR model, the inventory variable itself enters with a small, positive, and marginally insignificant coefficient ($\gamma = 0.0012$, $p = 0.06$), suggesting that variation in oil stocks does not independently explain realized currency volatility. More importantly, the interaction between the Brown Index and inventories is negative and statistically significant ($\eta = -0.0030$, $p = 0.007$), indicating that the predictive effect of the Brown Index is stronger when inventories are low. The conditional marginal effect of the Brown Index on realized volatility is given by:

$$\frac{\partial RV_{t+1}}{\partial Brown_t} = \delta + \eta Inv_t. \quad (21)$$

Evaluating this derivative at different levels of inventory tightness shows that the Brown effect is 0.0062 at the 25th percentile of inventories (tight markets) and 0.0038 at the 75th percentile (ample supply), implying roughly a 40% attenuation of the Brown effect when inventories are high.

The negative and statistically significant interaction between the Brown Index and oil inventories provides direct evidence for a commodity price and energy supply channel. The forecasting power of the Brown Index intensifies precisely when energy markets are tight, suggesting that it encapsulates information about both financial and physical vulnerabilities in the energy system. When inventories are low, supply shocks generate greater uncertainty about future energy availability, heightening the volatility of energy-sector equities and increasing the sensitivity of currency volatility to brown-sector dynamics. By contrast, when inventories are abundant, energy markets can better absorb shocks, dampening the volatility transmission from brown-sector firms to exchange rates.

These findings imply that the Brown Index does not merely proxy for global financial risk, but also embeds information about real-side energy market conditions. Its predictive ability arises from the combined influence of commodity price fluctuations, energy supply tightness, and their interaction with financial market expectations. Accordingly, the Brown Index can be interpreted as a composite measure of climate–energy–financial risk: it integrates signals from both the financial and physical dimensions of energy markets, capturing how structural constraints in energy supply amplify volatility transmission across asset classes.

5.2 Industrial Transition and Policy Uncertainty Channel

Another key mechanism linking the Brown Index to exchange rate volatility operates through the industrial transition and policy uncertainty channel. Broadly speaking, policy uncertainty affects multiple economic sectors (Baker et al. (2016)). More specifically, stricter climate policies are associated with higher risk premia globally (Bolton and Kacperczyk (2023)). In our setting, this channel reflects how the volatility of carbon-intensive firms responds to uncertainty surrounding climate transition policies,

regulatory frameworks, and technological substitution risks. Within the Brown portfolio, firms such as *Air Products and Chemicals (APD)* and *Albemarle (ALB)* are particularly illustrative. *APD* produces industrial gases that are crucial to fossil-fuel processing as well as to emerging hydrogen technologies, while *ALB* occupies a dual role: it is both a lithium supplier to electric-vehicle battery production and a mining-intensive firm whose valuation is influenced by commodity-price volatility. These firms are thus directly exposed to both sides of the energy transition—traditional carbon-intensive operations and emerging green technologies.

The valuation and volatility of such firms fluctuate with changes in environmental policy, carbon pricing schemes, and government subsidy programs. Periods of heightened uncertainty about climate regulation or policy direction can alter expected cash flows and discount rates for both brown and transition-exposed firms, increasing volatility in their returns. Accordingly, the Brown Index, which aggregates volatility from these firms, may embed information about uncertainty over the speed and nature of the industrial transition. To capture this dimension, we interact the Brown Index with the U.S. Climate Policy Uncertainty (CPU) Index developed by Gavriilidis (2021)³, which quantifies the level of uncertainty surrounding U.S. climate-related policies at monthly frequency.

5.2.1 Empirical Specification and Findings

To formally test this mechanism, we augment the HAR model of realized volatility by introducing the CPU index and its interaction with the Brown Index as follows:

$$RV_{t+1} = \alpha + \beta_w RV_t^w + \beta_m RV_t^m + \delta Brown_t + \gamma CPU_t + \eta(Brown_t \times CPU_t) + \varepsilon_{t+1}. \quad (22)$$

Here, CPU_t denotes the standardized value of the Climate Policy Uncertainty (CPU) Index. Because the CPU series is available at a monthly frequency, the HAR model is resampled to the same frequency to ensure temporal consistency across variables.

Table 16 reports results for the climate policy uncertainty channel using monthly data from 2015 to 2023. The benchmark HAR model augmented by the Brown Index (Model 1) indicates that brown-sector volatility remains a statistically significant predictor of realized currency volatility ($\delta = 0.0119$, $p = 0.002$). Adding the CPU index (Model 2) yields a small and statistically insignificant coefficient ($\gamma = -0.0024$, $p = 0.36$), suggesting that climate policy uncertainty does not directly explain realized volatility once the Brown Index is included. The Brown Index itself remains robust and significant ($\delta = 0.0126$, $p = 0.001$), and the model's R^2 increases only marginally (from 0.532 to 0.534).

³. CPU is the U.S. Climate Policy Uncertainty (CPU) Index from policyuncertainty.com/climate_uncertainty.html

Model 3 incorporates the interaction between the Brown Index and the CPU index to test whether policy uncertainty modifies the predictive effect of brown sector volatility. The estimated interaction coefficient is positive but statistically insignificant ($\eta = 0.0015$, $p = 0.63$), implying that the forecasting power of the Brown Index does not vary meaningfully across levels of climate policy uncertainty. At low levels of policy uncertainty (25th percentile of CPU), the marginal effect is 0.0107, while at high uncertainty (75th percentile), it is 0.0130, indicating only a modest and statistically insignificant increase in sensitivity under elevated policy uncertainty.

The results suggest that while the Brown Index continues to serve as a statistically significant predictor of exchange rate volatility, its forecasting strength is not materially affected by climate policy uncertainty. The lack of a significant interaction implies that the predictive relationship between brown-sector volatility and currency volatility is largely stable across different policy uncertainty regimes. This finding indicates that the Brown Index already incorporates much of the information about transition-related risks and regulatory uncertainty that the CPU index is designed to capture. In other words, the volatility of carbon intensive firms reflects expectations of industrial transition and policy adjustments endogenously, without requiring additional explanatory power from an external uncertainty index.

Taken together, the evidence provides limited support for a distinct transition-policy amplification channel. The Brown Index remains a strong predictor of realized volatility, but its predictive content does not vary systematically with climate policy uncertainty. This suggests that the Brown Index encapsulates broader market perceptions of transition and regulatory risk as part of its underlying structure. Rather than being a pure reflection of short-term policy uncertainty, the Brown Index functions as an integrated measure of industrial transition risk—aggregating information about carbon exposure, technological change, and investor expectations regarding the policy environment. The stability of its predictive power across varying policy uncertainty regimes underscores its robustness as a composite indicator of climate-related financial risk.

5.3 Global Risk Premium and Hedging Channel

A further mechanism connecting the Brown Index to exchange rate volatility operates through the global risk premium and hedging channel. Risk premia in global markets reflect a number of considerations, including volatility (Della Corte et al. (2016); Fang and Liu (2021)). In the present setting, carbon intensive firms are tightly linked to commodity derivatives markets. Their cash flows are directly exposed to oil and metals prices and these firms, their counterparties, and intermediaries engage in hedging using options and futures. When option dealers dynamically hedge their positions in energy markets, changes in oil prices can generate feedback effects through *gamma* and liquidity channels.

Such hedging activities influence not only commodity volatility but also foreign exchange (FX) volatility through crossmarket riskpremium adjustments, correlated trading flows, and shifts in global liquidity conditions. Therefore, the Brown Index may capture volatility spillovers that originate from commodity derivatives and propagate to currency markets via the pricing of global risk.

In this context, the VIX serves as a natural benchmark for global risk aversion and liquidity conditions. If the Brown Index captures risk spillovers that are distinct from those already embedded in the VIX, its coefficient should remain significant even after controlling for the VIX. Furthermore, if the Brown effect intensifies when global risk aversion rises, we would expect a positive and significant interaction term between the Brown Index and the VIX.

5.3.1 Empirical Specification and Findings

To evaluate this hypothesis, we estimate an augmented HAR model that incorporates both the Brown Index and the VIX as explanatory variables:

$$RV_{t+1} = \alpha + \beta_d RV_t^d + \beta_w RV_t^w + \beta_m RV_t^m + \delta Brown_t + \gamma VIX_t + \eta(Brown_t \times VIX_t) + \varepsilon_{t+1}, \quad (23)$$

VIX_t measures global risk aversion⁴. The interaction term $Brown_t \times VIX_t$ tests whether the predictive power of the Brown Index varies with global risk conditions. If the Brown Index captures hedging-driven volatility spillovers that intensify during global stress episodes, we expect $\eta > 0$.

Table 17 presents the results of the global risk channel analysis using daily data from 2015 to 2023. The baseline HAR model explains 55.9% of the variation in realized volatility. Adding the Brown Index improves the explanatory power to $R^2 = 0.565$, with a statistically significant coefficient ($\delta = 0.0050$, $p < 0.01$). This indicates that brown-sector volatility enhances the forecasting of currency volatility beyond standard HAR dynamics.

When the VIX is included, its coefficient is negative and insignificant ($\gamma = -0.0007$, $p = 0.66$), while the Brown Index remains significant ($\delta = 0.0056$, $p = 0.007$). The model's R^2 is unchanged at 0.565, implying that the VIX does not add independent explanatory power once brown-sector volatility is considered. Model 4 adds the interaction term $Brown_t \times VIX_t$ to test for regime dependence. The coefficient on the interaction is positive but insignificant ($\eta = 0.0004$, $p = 0.55$), suggesting that the effect of the Brown Index does not vary materially across high- and low-VIX regimes.

The results provide two main insights. First, the Brown Index retains strong and significant predictive power for realized exchange rate volatility even after controlling for the VIX, indicating that it captures risk transmission mechanisms not fully reflected in broad equity market risk measures. This

⁴The VIX data are obtained from Cboe Global Markets.

finding supports the interpretation of the Brown Index as encompassing volatility spillovers arising from dealer hedging, liquidity provision, and cross-market trading in commodity derivatives. Second, the absence of a significant interaction between the Brown Index and the VIX suggests that these spillovers are persistent rather than episodic. The predictive influence of brown sector volatility on currency markets is stable across both tranquil and stressed global environments, consistent with the notion that energy and commodity-linked volatility is a structural component of global risk pricing.

Overall, the results imply that the Brown Index integrates information from both financial and real markets, capturing hedging-related volatility propagation and risk premium adjustments that extend beyond conventional global risk indicators.

6 Conclusions

Does brown firms' volatility matter for predicting moves in currency and energy markets? Brown volatility is important since it may contain useful information about risk exposure in these markets. In a world riven by geopolitical uncertainty, such information is valuable for both investors and policymakers seeking a glimpse of future trends.

But brown firms' information arises at multiple frequencies, none of which is necessarily most important. To address our question, therefore, we build brown and green portfolios at multiple frequencies, based on intraday stock price data. Thence we compute realized volatility at daily, weekly, and monthly frequencies, and use the mixed-frequency method of Peng et al. (2024) to extract realized volatility indices for brown and green firms. We also compute realized volatility for currency and oil markets at multiple frequencies. As a benchmark, we apply HAR models to forecast the volatility of both currencies and oil prices. To this benchmark we add our brown and green volatility indices. The brown volatility index significantly enhances the HAR model's predictive ability more than the green index. Thus, in our sample brown volatility is an important predictor of currency and oil volatility.

When we examine economic channels that link brown-firm volatility to broader market fluctuations, we uncover three salient results. First is a channel for commodity prices and energy supply. Here we find that the sensitivity of realized volatility to the brown index increases when oil inventories are low, consistent with volatility transmission through energy market tightness. Second is a channel for the industrial transition and policy uncertainty. Here we discover that, while policy uncertainty itself adds limited explanatory power, the brown index remains a statistically significant predictor—suggesting that it possesses information about transition-related risks. Third is a channel for the global risk premium and hedging. Here, when we control for VIX, the brown index continues to exhibit significant in-sample explanatory power, indicating that it captures risk-transmission mechanisms not fully reflected in stan-

dard global risk indicators. From this perspective, brown firms' volatility appears to convey a composite signal of climate–energy–financial risk. In sum, our index aggregates information about commodity market stress, transition uncertainty, and global risk premia into a single measure, which improves forecasting performance for volatility of both the oil market and exchange rates.

Our results complement and build on recent work highlighting the value of climate and macro risks for currency forecasting (Bonato et al. (2023)) and for energy forecasting (Guidolin and Pedio (2021), Rao et al. (2023)). The results also complement recent research on climate-related expectations in derivatives, manufacturing, and international settings (Schlenker and Taylor (2021), Downey et al. (2023), and Hale (2024)). Just as our project was invigorated and inspired by previous research, future work can fruitfully develop our inroad and apply our multi-frequency indices to asset allocation and modeling in other international and energy markets.

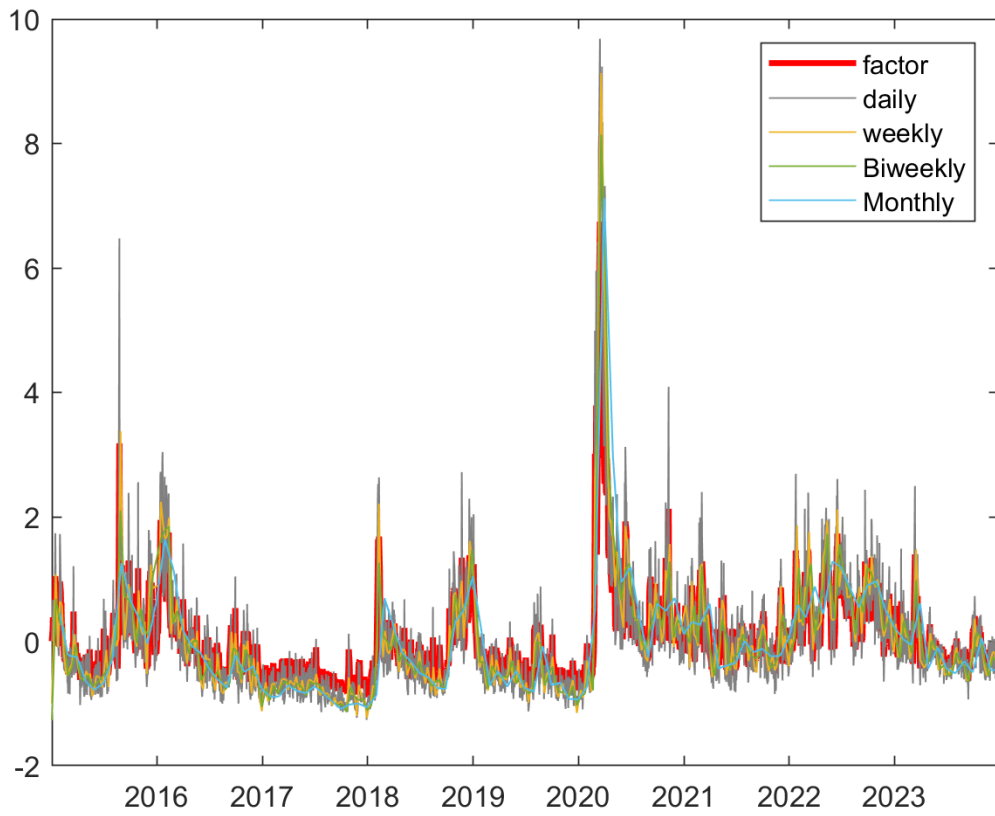
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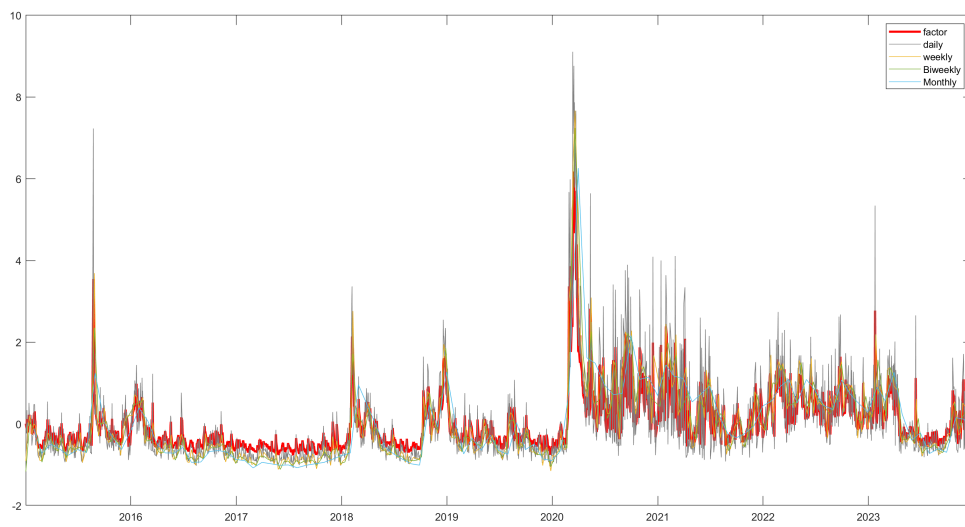
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Figure 1: The Brown Factor



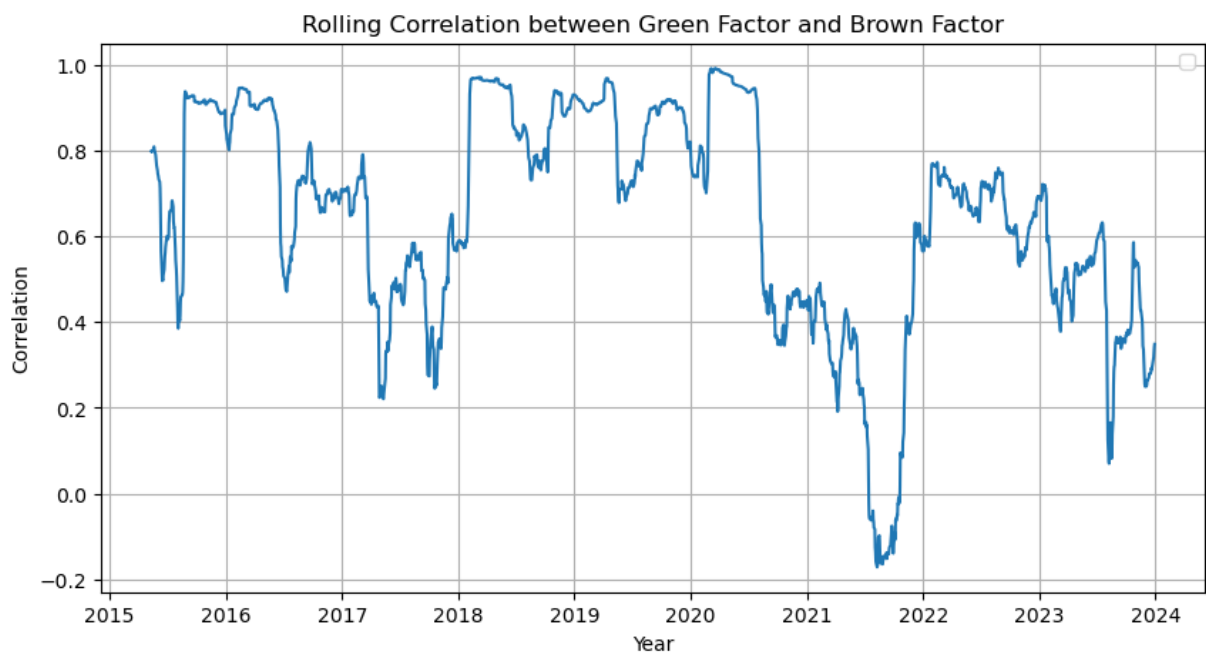
The figure shows the Brown Factor, computed using the mixed-frequency approach as in Peng et al. (2024).

Figure 2: The Green Factor



The figure shows the Green Factor, computed using the mixed-frequency approach as in Peng et al. (2024).

Figure 3: 60-Day Rolling Correlations of Brown and Green Indices



The

figure shows the rolling correlations of the brown and green indices.

Figure 4: Multiple-Day Rolling Correlations of Brown and Green Indices

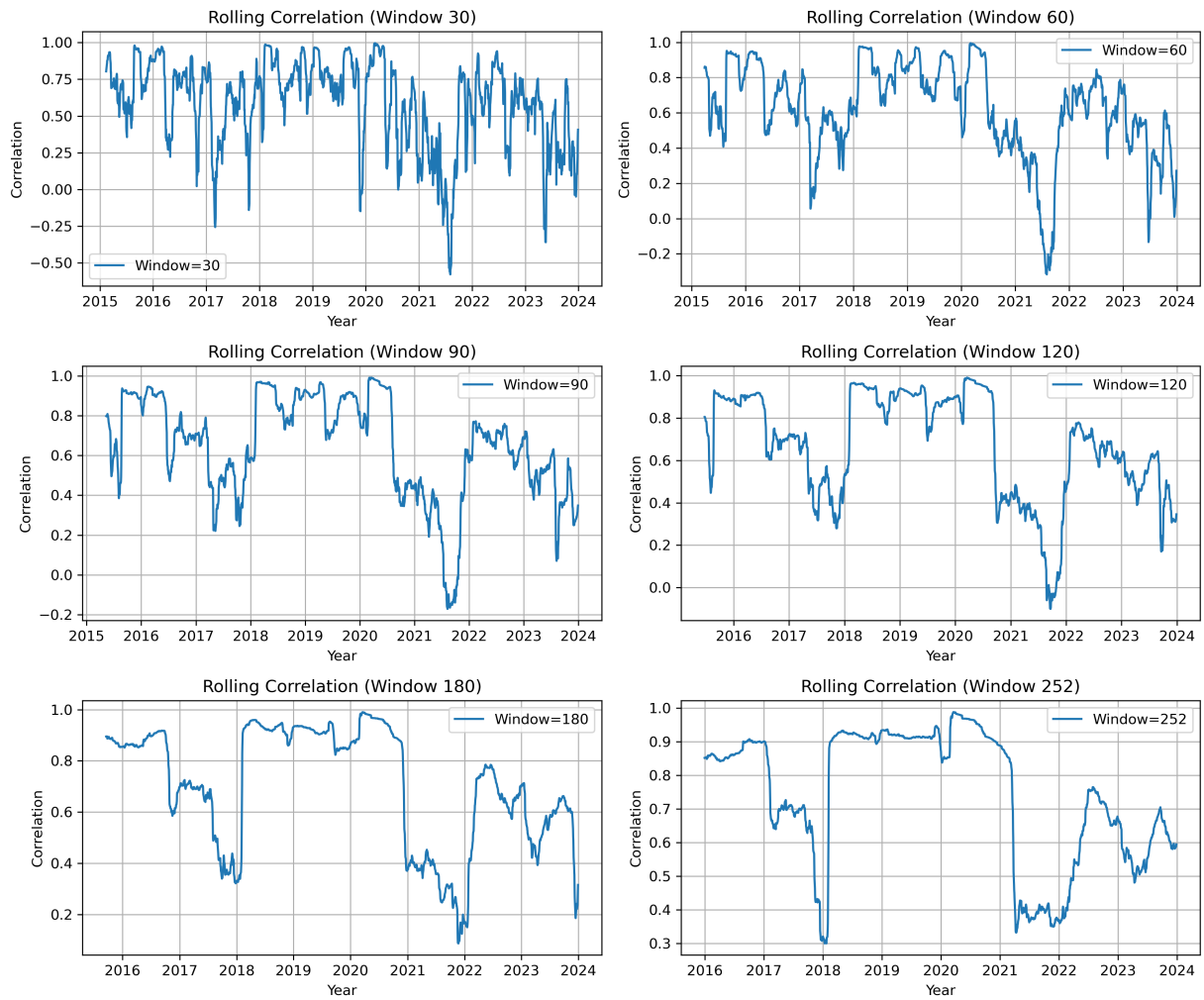


Table 1: Green and Brown Portfolios Composition

Sector	Firms	Count	Weight (%)
<i>Brown Portfolio</i>			
Oil & Gas	CVX, MPC, VLO, PSX, WMB	5	60.4
Metals & Mining	FCX, SCCO, NEM	3	14.4
Utilities / Power	D	1	9.7
Chemicals / Materials	APD, ALB	2	9.5
Consumer Diversifier	MNST	1	6.1
<i>Green Portfolio</i>			
Health Care	UNH, ABT, ELV, CI	4	31.9
Financials	V, AXP, BLK	3	29.0
Communication Services	DIS, VZ	2	31.6
Professional Services	ADP, MMC	2	7.5

Notes. Sector weights are calculated as the sum of firm-level market capitalization proportions within each category. “Brown Portfolio” represents high-carbon or resource-intensive sectors (e.g., energy, mining, utilities, and materials), while “Green Portfolio” includes lower-carbon, service-oriented, and health-focused firms. Weights are expressed as percentages of each portfolio’s total market capitalization.

Table 2: Summary Statistics

Asset	Mean	Std. Dev.	Skewness	Kurtosis	N
AUD	2.88E-07	0.0004	1.0627	123.8235	614739
EUR	1.43E-07	0.0003	-0.4795	108.1160	614829
CAD	2.09E-07	0.0003	0.2800	73.2842	615153
NOK	4.70E-07	0.0005	2.0090	276.2583	611394
WTI	1.51E-06	0.0020	1.6205	781.3678	547550
Brent	1.25E-06	0.0017	1.2100	216.1800	516144
Green Index	0.00016	0.6925	2.9075	15.1705	2264
Brown Index	0.00013	0.7199	3.8859	25.9370	2264

Notes. The table reports the mean, standard deviation, skewness, and kurtosis of the returns for foreign exchange (FX) rates and oil prices including AUD, EUR, CAD, NOK, WTI and Brent. It also presents the corresponding statistics for the Green and Brown Indexes. “N” denotes the number of observations.

Table 3: Forecasting Performance of Green and Brown Indices, Plus OVX

The table presents forecast performance as described in Section 3. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro. HAR denotes the heterogeneous autoregressive model (Corsi (2009)). OVX denotes the Oil Volatility Index from CBOE. We utilize a rolling window of 600 days to forecast one-day-ahead realized volatility. The forecast period runs from June 22, 2017 to November 30, 2023.

Exchange Rate	Model	RMSE	Out of Sample R^2
AUD	<i>HAR</i>	0.0375	0.3503
	<i>HAR+Green Factor</i>	0.0373	0.3596
	<i>HAR+Brown Factor</i>	0.0369	0.3718
	<i>HAR+OVX</i>	0.0378	0.3460
	<i>HAR+Green Factor+Brown Factor</i>	0.0371	0.3670
EUR	<i>HAR</i>	0.0235	0.4042
	<i>HAR+Green Factor</i>	0.0235	0.4020
	<i>HAR+Brown Factor</i>	0.0235	0.4058
	<i>HAR+OVX</i>	0.0237	0.3978
	<i>HAR+Green Factor+Brown Factor</i>	0.0235	0.4024
CAD	<i>HAR</i>	0.0221	0.3127
	<i>HAR+Green Factor</i>	0.0219	0.3187
	<i>HAR+Brown Factor</i>	0.0217	0.3241
	<i>HAR+OVX</i>	0.0221	0.3049
	<i>HAR+Green Factor+Brown Factor</i>	0.0219	0.3205
NOK	<i>HAR</i>	0.0490	0.3750
	<i>HAR+Green Factor</i>	0.0490	0.3910
	<i>HAR+Brown Factor</i>	0.0480	0.4056
	<i>HAR+OVX</i>	0.0490	0.3823
	<i>HAR+Green Factor+Brown Factor</i>	0.0482	0.3986
WTI	<i>HAR</i>	0.3833	0.2077
	<i>HAR+Green Factor</i>	0.3846	0.2026
	<i>HAR+Brown Factor</i>	0.3828	0.2098
	<i>HAR+OVX</i>	0.3624	0.2917
	<i>HAR+Green Factor+Brown Factor</i>	0.3822	0.2121
BRENT	<i>HAR</i>	0.1552	0.5534
	<i>HAR+Green Factor</i>	0.1556	0.5522
	<i>HAR+Brown Factor</i>	0.1549	0.5558
	<i>HAR+OVX</i>	0.1425	0.6251
	<i>HAR+Green Factor+Brown Factor</i>	0.1556	0.5524

Table 4: Forecasting Performance with Machine Learning Framework

The table presents forecast performance with a Long Short Term Memory (LSTM) framework, as described in Section 3. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro. OVX denotes the Oil Volatility Index from CBOE. LSTM denotes the long short term memory model. The forecasting horizon is 1 day. The forecast period is July 9, 2019 to November 30, 2023.

Exchange Rate	Model	RMSE	Out of Sample R^2
AUD	<i>LSTM</i>	0.0413	0.3518
	<i>LSTM+Green Factor</i>	0.0402	0.3863
	<i>LSTM+Brown Factor</i>	0.0395	0.4066
	<i>LSTM+OVX</i>	0.0415	0.3454
	<i>LSTM+Green Factor+Brown Factor</i>	0.0400	0.3917
EUR	<i>LSTM</i>	0.0263	0.4141
	<i>LSTM+Green Factor</i>	0.0264	0.4120
	<i>LSTM+Brown Factor</i>	0.0263	0.4152
	<i>LSTM+OVX</i>	0.0264	0.4101
	<i>LSTM+Green Factor+Brown Factor</i>	0.0270	0.3832
CAD	<i>LSTM</i>	0.0226	0.3709
	<i>LSTM+Green Factor</i>	0.0224	0.3860
	<i>LSTM+Brown Factor</i>	0.0223	0.3882
	<i>LSTM+OVX</i>	0.0233	0.3348
	<i>LSTM+Green Factor+Brown Factor</i>	0.0226	0.3727
NOK	<i>LSTM</i>	0.0550	0.3850
	<i>LSTM+Green Factor</i>	0.0538	0.4116
	<i>LSTM+Brown Factor</i>	0.0528	0.4203
	<i>LSTM+OVX</i>	0.0550	0.3860
	<i>LSTM+Green Factor+Brown Factor</i>	0.0537	0.4139
WTI	<i>LSTM</i>	0.4506	0.2082
	<i>LSTM+Green Factor</i>	0.4497	0.2116
	<i>LSTM+Brown Factor</i>	0.4488	0.2161
	<i>LSTM+OVX</i>	0.4468	0.2229
	<i>LSTM+Green Factor+Brown Factor</i>	0.4491	0.2150
BRENT	<i>LSTM</i>	0.1788	0.5547
	<i>LSTM+Green Factor</i>	0.1793	0.5522
	<i>LSTM+Brown Factor</i>	0.1778	0.5595
	<i>LSTM+OVX</i>	0.1766	0.5655
	<i>LSTM+Green Factor+Brown Factor</i>	0.1806	0.5456

Table 5: **Out-of-Sample Performance Evaluated by MCS Test**

This table reports the MCS test results of various forecasting models. Each entry is a p-value based on T_R or T_{SQ} statistics. The threshold value is 0.25. P-value = 1 is shown in bold and underlined. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro.

Exchange Rate	Model	T_R	T_{SQ}
AUD	<i>HAR</i>	0.8432	0.8586
	<i>HAR+Green Factor</i>	0.0172	0.0258
	<i>HAR+Brown Factor</i>	1.0000	1.0000
	<i>HAR+OVX</i>	0.0550	0.3062
	<i>HAR+Green Factor+Brown Factor</i>	0.0550	0.3062
EUR	<i>HAR</i>	0.7478	0.7432
	<i>HAR+Green Factor</i>	0.3538	0.4682
	<i>HAR+Brown Factor</i>	1.0000	1.0000
	<i>HAR+OVX</i>	0.2788	0.3384
	<i>HAR+Green Factor+Brown Factor</i>	0.3538	0.5200
CAD	<i>HAR</i>	0.0442	0.0406
	<i>HAR+Green Factor</i>	0.0000	0.0006
	<i>HAR+Brown Factor</i>	1.0000	1.0000
	<i>HAR+OVX</i>	0.2156	0.3678
	<i>HAR+Green Factor+Brown Factor</i>	0.2156	0.3678
NOK	<i>HAR</i>	0.6544	0.6574
	<i>HAR+Green Factor</i>	0.0864	0.1306
	<i>HAR+Brown Factor</i>	1.0000	1.0000
	<i>HAR+OVX</i>	0.0864	0.3140
	<i>HAR+Green Factor+Brown Factor</i>	0.0864	0.3140
WTI	<i>HAR</i>	0.6600	0.5824
	<i>HAR+Green Factor</i>	0.0394	0.1524
	<i>HAR+Brown Factor</i>	0.6600	0.5824
	<i>HAR+OVX</i>	1.0000	1.0000
	<i>HAR+Green Factor+Brown Factor</i>	0.0104	0.0302
BRENT	<i>HAR</i>	0.0685	0.2790
	<i>HAR+Green Factor</i>	0.0682	0.2790
	<i>HAR+Brown Factor</i>	1.0000	1.0000
	<i>HAR+OVX</i>	0.0682	0.2790
	<i>HAR+Green Factor+Brown Factor</i>	0.3490	0.3528

Table 6: **Out-of-Sample Performance Evaluated by MCS Test**

Exchange Rate	Model	T_R	T_{SQ}
AUD	<i>LSTM</i>	0.5040	0.6396
	<i>LSTM+Green Factor</i>	0.5040	0.6396
	<i>LSTM+Brown Factor</i>	1.0000	1.0000
	<i>LSTM+OVX</i>	0.1624	0.3650
	<i>LSTM+Green Factor+Brown Factor</i>	0.5040	0.6396
EUR	<i>LSTM</i>	0.8742	0.8680
	<i>LSTM+Green Factor</i>	0.8308	0.8654
	<i>LSTM+Brown Factor</i>	1.0000	1.0000
	<i>LSTM+OVX</i>	0.3904	0.5976
	<i>LSTM+Green Factor+Brown Factor</i>	0.1542	0.1698
CAD	<i>LSTM</i>	0.0808	0.0716
	<i>LSTM+Green Factor</i>	0.0808	0.0716
	<i>LSTM+Brown Factor</i>	1.0000	1.0000
	<i>LSTM+OVX</i>	0.0130	0.0486
	<i>LSTM+Green Factor+Brown Factor</i>	0.0130	0.0486
NOK	<i>LSTM</i>	0.7922	0.6540
	<i>LSTM+Green Factor</i>	0.7922	0.7194
	<i>LSTM+Brown Factor</i>	1.0000	1.0000
	<i>LSTM+OVX</i>	0.7922	0.6540
	<i>LSTM+Green Factor+Brown Factor</i>	0.7922	0.7194
WTI	<i>LSTM</i>	0.3446	0.4044
	<i>LSTM+Green Factor</i>	0.3446	0.4192
	<i>LSTM+Brown Factor</i>	0.3446	0.4192
	<i>LSTM+OVX</i>	1.0000	1.0000
	<i>LSTM+Green Factor+Brown Factor</i>	0.3446	0.4192
BRENT	<i>LSTM</i>	0.7144	0.7918
	<i>LSTM+Green Factor</i>	0.5680	0.7140
	<i>LSTM+Brown Factor</i>	0.8752	0.8868
	<i>LSTM+OVX</i>	1.0000	1.0000
	<i>LSTM+Green Factor+Brown Factor</i>	0.5646	0.5210

Note*: This table reports the MCS test results of various forecasting models. Each entry is a p-value based on T_R or T_{SQ} statistics. The threshold value is 0.25. P-value = 1 is shown in bold and underlined. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro.

Table 7: Forecasting Performance of HAR Models in Low and High Volatility Regimes Based on VIX Values

Exchange Rate	Model	Out-of-Sample R^2		RMSE	
		High Vol	Low Vol	High Vol	Low Vol
USDAUD	HAR	0.2657	0.1566	0.0476	0.0239
	HAR+Green Factor	0.2811	0.1464	0.0471	0.0241
	HAR+Brown Factor	0.2951	0.1615	0.0466	0.0239
	HAR+OVX	0.2608	0.1509	0.0477	0.0240
	HAR+Green Factor+Brown Factor	0.2885	0.1602	0.0468	0.0239
USDEUR	HAR	0.3575	0.2481	0.0273	0.0188
	HAR+Green Factor	0.3544	0.2468	0.0274	0.0188
	HAR+Brown Factor	0.3568	0.2561	0.0274	0.0187
	HAR+OVX	0.3474	0.2478	0.0276	0.0188
	HAR+Green Factor+Brown Factor	0.3538	0.2501	0.0274	0.0188
USDCAD	HAR	0.277	0.2189	0.0237	0.0202
	HAR+Green Factor	0.2792	0.2317	0.0237	0.0200
	HAR+Brown Factor	0.2858	0.2364	0.0236	0.0200
	HAR+OVX	0.2577	0.2263	0.024	0.0201
	HAR+Green Factor+Brown Factor	0.2797	0.2357	0.0237	0.0200
USDNOK	HAR	0.2677	0.2126	0.0636	0.0287
	HAR+Green Factor	0.2906	0.2101	0.0626	0.0287
	HAR+Brown Factor	0.3089	0.2221	0.0618	0.0285
	HAR+OVX	0.2778	0.2133	0.0632	0.0287
	HAR+Green Factor+Brown Factor	0.3009	0.2217	0.0622	0.0285
WTI	HAR	0.1614	0.0116	0.5233	0.1415
	HAR+Green Factor	0.1553	0.0155	0.5252	0.1413
	HAR+Brown Factor	0.1623	0.035	0.523	0.1399
	HAR+OVX	0.2538	0.0605	0.4936	0.1380
	HAR+Green Factor+Brown Factor	0.1658	0.0200	0.5219	0.1409
BRENT	HAR	0.4904	0.4516	0.2051	0.0788
	HAR+Green Factor	0.489	0.450	0.2053	0.0789
	HAR+Brown Factor	0.492	0.4623	0.2047	0.0780
	HAR+OVX	0.582	0.4683	0.1857	0.0776
	HAR+Green Factor+Brown Factor	0.4882	0.4577	0.2055	0.0783

Note*: Table 6 reports the out-of-sample R^2 and the root mean square error (RMSE) of the HAR type forecasting models in high volatility and low volatility regimes. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro.

Table 8: Forecasting Performance of LSTM Models in Low and High Volatility Regimes Based on VIX Values

Exchange Rate	Model	Out-of-Sample R^2		RMSE	
		High Vol	Low Vol	High Vol	Low Vol
USDAUD	LSTM	0.2218	0.2499	0.0512	0.028
	LSTM+Green Factor	0.2939	0.1915	0.0488	0.0291
	LSTM+Brown Factor	0.3115	0.2365	0.0482	0.0283
	LSTM+OVX	0.2123	0.2483	0.0515	0.0281
	LSTM+Green Factor+Brown Factor	0.3008	0.1963	0.0485	0.029
USDEUR	LSTM	0.3208	0.1859	0.0302	0.0218
	LSTM+Green Factor	0.318	0.184	0.0303	0.0218
	LSTM+Brown Factor	0.3255	0.1795	0.0301	0.0218
	LSTM+OVX	0.3111	0.192	0.0304	0.0217
	LSTM+Green Factor+Brown Factor	0.2751	0.1658	0.0312	0.022
USDCAD	LSTM	0.2431	0.1924	0.026	0.0186
	LSTM+Green Factor	0.274	0.1852	0.0255	0.0187
	LSTM+Brown Factor	0.2691	0.2038	0.0256	0.0186
	LSTM+OVX	0.1666	0.2147	0.0273	0.0184
	LSTM+Green Factor+Brown Factor	0.2566	0.1714	0.0258	0.0189
USDNOK	LSTM	0.2585	0.2432	0.0694	0.0353
	LSTM+Green Factor	0.3114	0.1939	0.0669	0.0364
	LSTM+Brown Factor	0.3183	0.2186	0.0665	0.0359
	LSTM+OVX	0.2571	0.2549	0.0695	0.035
	LSTM+Green Factor+Brown Factor	0.3043	0.2355	0.0672	0.0355
WTI	LSTM	0.1618	0.0869	0.6142	0.172
	LSTM+Green Factor	0.1659	0.0858	0.6127	0.1721
	LSTM+Brown Factor	0.1732	0.0565	0.61	0.1751
	LSTM+OVX	0.1814	0.0488	0.607	0.1758
	LSTM+Green Factor+Brown Factor	0.1722	0.0521	0.6104	0.1755
BRENT	LSTM	0.4627	0.571	0.2352	0.0928
	LSTM+Green Factor	0.4606	0.5642	0.2357	0.0935
	LSTM+Brown Factor	0.4696	0.5703	0.2337	0.0929
	LSTM+OVX	0.5332	0.2862	0.2192	0.1197
	LSTM+Green Factor+Brown Factor	0.4501	0.57	0.238	0.0929

Note*: Table 7 reports the out-of-sample R^2 and the root mean square error (RMSE) of the LSTM type forecasting models in high volatility and low volatility regimes. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro.

Table 9: Forecasting Performance of HAR Models in Low and High Volatility Regimes Based on Regime Switching Model

Exchange Rate	Model	Out-of-Sample R^2		RMSE	
		High Vol	Low Vol	High Vol	Low Vol
USDAUD	HAR	0.2062	0.1839	0.0544	0.0256
	HAR+Green Factor	0.2264	0.1753	0.0537	0.0257
	HAR+Brown Factor	0.2403	0.1929	0.0532	0.0255
	HAR+OVX	0.1991	0.1823	0.0546	0.0256
	HAR+Green Factor+Brown Factor	0.2322	0.1917	0.0535	0.0255
USDEUR	HAR	0.3068	0.2465	0.0296	0.0198
	HAR+Green Factor	0.3013	0.2472	0.0297	0.0198
	HAR+Brown Factor	0.3048	0.2532	0.0296	0.0197
	HAR+OVX	0.2903	0.2493	0.0299	0.0197
	HAR+Green Factor+Brown Factor	0.2987	0.2515	0.0298	0.0197
USDCAD	HAR	0.2521	0.2164	0.0255	0.0201
	HAR+Green Factor	0.2584	0.2234	0.0254	0.0200
	HAR+Brown Factor	0.2640	0.2297	0.0253	0.0199
	HAR+OVX	0.2299	0.2190	0.0259	0.0200
	HAR+Green Factor+Brown Factor	0.2554	0.2295	0.0255	0.0199
USDNOK	HAR	0.1753	0.2325	0.0738	0.0307
	HAR+Green Factor	0.2058	0.2265	0.0725	0.0308
	HAR+Brown Factor	0.2248	0.2451	0.0716	0.0304
	HAR+OVX	0.1865	0.2368	0.0733	0.0306
	HAR+Green Factor+Brown Factor	0.2153	0.2431	0.0720	0.0305
WTI	HAR	0.1405	0.0736	0.6302	0.1519
	HAR+Green Factor	0.1338	0.0779	0.6327	0.1516
	HAR+Brown Factor	0.1403	0.0985	0.6303	0.1499
	HAR+OVX	0.2338	0.1518	0.5950	0.1454
	HAR+Green Factor+Brown Factor	0.1444	0.0862	0.6288	0.1509
BRENT	HAR	0.4616	0.4704	0.2368	0.0911
	HAR+Green Factor	0.4598	0.4699	0.2372	0.0912
	HAR+Brown Factor	0.3943	0.5401	0.3033	0.1038
	HAR+OVX	0.5613	0.5117	0.2138	0.0875
	HAR+Green Factor+Brown Factor	0.4573	0.4793	0.2378	0.0904

Note*: Table 8 reports the out-of-sample R^2 and the root mean square error (RMSE) of the HAR type forecasting models in high volatility and low volatility regimes. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro.

Table 10: Forecasting Performance of LSTM Models in Low and High Volatility Regimes Based on Regime Switching Model

Exchange Rate	Model	Out-of-Sample R^2		RMSE	
		High Vol	Low Vol	High Vol	Low Vol
USDAUD	LSTM	0.2197	0.2147	0.0520	0.0293
	LSTM+Green Factor	0.2976	0.1598	0.0494	0.0303
	LSTM+Brown Factor	0.3146	0.2041	0.0488	0.0295
	LSTM+OVX	0.2094	0.2136	0.0524	0.0294
	LSTM+Green Factor+Brown Factor	0.3022	0.1712	0.0492	0.0301
USDEUR	LSTM	0.3086	0.1653	0.0302	0.0225
	LSTM+Green Factor	0.3059	0.1629	0.0303	0.0226
	LSTM+Brown Factor	0.3133	0.1608	0.0301	0.0226
	LSTM+OVX	0.2978	0.1707	0.0305	0.0225
	LSTM+Green Factor+Brown Factor	0.2567	0.1495	0.0314	0.0228
USDCAD	LSTM	0.2634	0.2017	0.0258	0.0196
	LSTM+Green Factor	0.2985	0.1934	0.0252	0.0197
	LSTM+Brown Factor	0.2896	0.2145	0.0253	0.0194
	LSTM+OVX	0.1824	0.2172	0.0272	0.0194
	LSTM+Green Factor+Brown Factor	0.2807	0.1802	0.0255	0.0198
USDNOK	LSTM	0.2527	0.2126	0.0710	0.0366
	LSTM+Green Factor	0.3081	0.1698	0.0683	0.0376
	LSTM+Brown Factor	0.3143	0.1953	0.0680	0.0370
	LSTM+OVX	0.2504	0.2256	0.0711	0.0363
	LSTM+Green Factor+Brown Factor	0.2993	0.2114	0.0688	0.0367
WTI	LSTM	0.1592	0.0717	0.6393	0.1722
	LSTM+Green Factor	0.1630	0.0741	0.6379	0.1720
	LSTM+Brown Factor	0.1700	0.0511	0.6352	0.1744
	LSTM+OVX	0.1776	0.0538	0.6323	0.1741
	LSTM+Green Factor+Brown Factor	0.1688	0.0495	0.6356	0.1745
BRENT	LSTM	0.4629	0.5411	0.2417	0.0987
	LSTM+Green Factor	0.4607	0.5350	0.2422	0.0994
	LSTM+Brown Factor	0.4680	0.5489	0.2405	0.0979
	LSTM+OVX	0.5341	0.3015	0.2251	0.1218
	LSTM+Green Factor+Brown Factor	0.4497	0.5409	0.2446	0.0988

Note*: Table 9 reports the out-of-sample R^2 and the root mean square error (RMSE) of the LSTM type forecasting models in high volatility and low volatility regimes based on regime switching model. AUD, CAD, NOK, and EUR denote the Australian dollar; Canadian dollar; Norwegian kroner; and the Euro.

Table 11: MCS Test Results for HAR Model in Low and High Volatility Regimes Based on VIX Values

Exchange Rate	Model	High Volatility		Low Volatility	
		T_R	T_{SQ}	T_R	T_{SQ}
USDAUD	HAR	0.5456	0.4688	0.6550	0.6480
	HAR+Green Factor	0.5456	0.4688	0.3436	0.2966
	HAR+Brown Factor	1.0000	1.0000	1.0000	1.0000
	HAR+OVX	0.5456	0.4622	0.4512	0.4390
	HAR+Green+Brown Factors	0.5456	0.4688	0.6550	0.6480
USDEUR	HAR	1.0000	1.0000	0.1290	0.1836
	HAR+Green Factor	0.8572	0.8216	0.1290	0.1836
	HAR+Brown Factor	0.9354	0.9364	1.0000	1.0000
	HAR+OVX	0.5328	0.5724	0.1290	0.1836
	HAR+Green+Brown Factors	0.8572	0.8216	0.1290	0.1836
USDCAD	HAR	0.4988	0.6904	0.0128	0.0112
	HAR+Green Factor	0.4988	0.6904	0.5542	0.4510
	HAR+Brown Factor	1.0000	1.0000	1.0000	1.0000
	HAR+OVX	0.3614	0.4068	0.2768	0.2400
	HAR+Green+Brown Factors	0.4988	0.6904	0.5542	0.5224
USDNOK	HAR	0.3262	0.2730	0.6324	0.4996
	HAR+Green Factor	0.3262	0.2730	0.6324	0.4996
	HAR+Brown Factor	1.0000	1.0000	1.0000	1.0000
	HAR+OVX	0.3262	0.2730	0.6324	0.4996
	HAR+Green+Brown Factors	0.3262	0.2730	0.9054	0.9038
WTI	HAR	0.4432	0.3598	0.1660	0.1220
	HAR+Green Factor	0.4432	0.3398	0.1272	0.0594
	HAR+Brown Factor	0.4432	0.3598	0.1660	0.1220
	HAR+OVX	1.0000	1.0000	1.0000	1.0000
	HAR+Green+Brown Factors	0.4432	0.3598	0.0036	0.0220
BRENT (BCO)	HAR	0.1756	0.2880	0.4722	0.5266
	HAR+Green Factor	0.0462	0.1584	0.4722	0.5266
	HAR+Brown Factor	0.7510	0.7536	0.4722	0.5266
	HAR+OVX	1.0000	1.0000	1.0000	1.0000
	HAR+Green+Brown Factors	0.3288	0.4622	0.4722	0.5266

Note: This table reports the MCS (Model Confidence Set) test results of Hansen et al. (2011)— T_R and T_{SQ} statistics—for HAR models under high and low volatility regimes identified by VIX. All values are rounded to four decimal places. Higher values indicate stronger performance.

Table 12: MCS Test Results for LSTM Model in Low and High Volatility Regimes Based on VIX Values

Exchange Rate	Model	High Volatility		Low Volatility	
		T_R	T_{SQ}	T_R	T_{SQ}
USDAUD	LSTM	0.8080	0.7424	1.0000	1.0000
	LSTM+Green Factor	0.8080	0.7944	0.0168	0.0088
	LSTM+Brown Factor	1.0000	1.0000	0.0168	0.0088
	LSTM+OVX	0.1452	0.3866	0.6780	0.6788
	LSTM+Green+Brown Factors	0.8080	0.7944	0.0168	0.0082
USDEUR	LSTM	0.6894	0.7262	0.1434	0.1574
	LSTM+Green Factor	0.6842	0.7262	0.1434	0.1574
	LSTM+Brown Factor	1.0000	1.0000	0.1434	0.1532
	LSTM+OVX	0.1268	0.2804	1.0000	1.0000
	LSTM+Green+Brown Factors	0.1268	0.1430	0.0212	0.0312
USDCAD	LSTM	0.7028	0.6206	0.0178	0.0568
	LSTM+Green Factor	1.0000	1.0000	0.0178	0.0568
	LSTM+Brown Factor	0.7864	0.7828	0.3416	0.3288
	LSTM+OVX	0.1596	0.1914	1.0000	1.0000
	LSTM+Green+Brown Factors	0.7028	0.6206	0.0178	0.0568
USDNOK	LSTM	0.6778	0.5694	0.0796	0.0790
	LSTM+Green Factor	0.7326	0.7396	0.0214	0.0028
	LSTM+Brown Factor	1.0000	1.0000	0.0214	0.0082
	LSTM+OVX	0.6778	0.5694	1.0000	1.0000
	LSTM+Green+Brown Factors	0.6778	0.6378	0.0526	0.0432
WTI	LSTM	0.7232	0.6282	1.0000	1.0000
	LSTM+Green Factor	1.0000	1.0000	0.9112	0.9112
	LSTM+Brown Factor	0.7792	0.7990	0.0852	0.0780
	LSTM+OVX	0.1524	0.1946	0.1610	0.1342
	LSTM+Green+Brown Factors	0.7232	0.6282	0.1040	0.0920
BRENT (BCO)	LSTM	0.4416	0.3154	1.0000	1.0000
	LSTM+Green Factor	0.4416	0.3154	0.5562	0.5992
	LSTM+Brown Factor	0.4416	0.3154	0.9716	0.9772
	LSTM+OVX	1.0000	1.0000	0.0264	0.0198
	LSTM+Green+Brown Factors	0.4416	0.2480	0.9716	0.9772

Note: This table reports the MCS (Model Confidence Set) test results of Hansen et al. (2011)— T_R and T_{SQ} statistics—for LSTM models under high and low volatility regimes identified by VIX. Higher values indicate stronger performance.

Table 13: MCS Test Results for HAR Model in Low and High Volatility Regimes Based on Regime Switching Model

Exchange Rate	Model	High Volatility		Low Volatility	
		T_R	T_{SQ}	T_R	T_{SQ}
USDAUD	HAR	0.6068	0.5912	0.3482	0.2986
	HAR+Green Factor	0.6068	0.5912	0.0794	0.1140
	HAR+Brown Factor	1.0000	1.0000	1.0000	1.0000
	HAR+OVX	0.6068	0.5404	0.3482	0.2712
	HAR+Green+Brown Factors	0.6068	0.5912	0.5624	0.5626
USDEUR	HAR	1.0000	1.0000	0.4402	0.3578
	HAR+Green Factor	0.6046	0.5728	0.4402	0.3578
	HAR+Brown Factor	0.8468	0.8416	1.0000	1.0000
	HAR+OVX	0.3062	0.3280	0.6184	0.5838
	HAR+Green+Brown Factors	0.5592	0.5544	0.6184	0.5838
USDCAD	HAR	0.4640	0.7048	0.0676	0.0412
	HAR+Green Factor	0.5794	0.7048	0.2322	0.1972
	HAR+Brown Factor	1.0000	1.0000	1.0000	1.0000
	HAR+OVX	0.4640	0.4762	0.2018	0.1128
	HAR+Green+Brown Factors	0.4640	0.7048	0.8672	0.8702
USDNOK	HAR	0.3608	0.3294	0.4388	0.3308
	HAR+Green Factor	0.1206	0.1638	0.4388	0.3308
	HAR+Brown Factor	1.0000	1.0000	1.0000	1.0000
	HAR+OVX	0.4488	0.4482	0.4388	0.3308
	HAR+Green+Brown Factors	0.4748	0.4770	0.4388	0.3308
WTI	HAR	0.0320	0.0118	0.4848	0.3766
	HAR+Green Factor	0.0320	0.0078	0.4848	0.3652
	HAR+Brown Factor	0.0320	0.0160	0.4848	0.3766
	HAR+OVX	1.0000	1.0000	1.0000	1.0000
	HAR+Green+Brown Factors	0.0062	0.0056	0.4848	0.3766
BRENT (BCO)	HAR	0.5630	0.5880	0.1216	0.1774
	HAR+Green Factor	0.5630	0.5880	0.0362	0.0930
	HAR+Brown Factor	0.5630	0.5880	0.5556	0.4290
	HAR+OVX	1.0000	1.0000	1.0000	1.0000
	HAR+Green+Brown Factors	0.5630	0.5880	0.5556	0.4290

Note: This table reports the MCS (Model Confidence Set) test statistics of Hansen et al. (2011)— T_R and T_{SQ} —for HAR models across exchange rates and commodity prices, under high and low volatility regimes as determined by a regime switching model. All values are rounded to four decimal places.

Table 14: MCS Test Results for LSTM Model in Low and High Volatility Regimes Based on Regime Switching Model

Exchange Rate	Model	High Volatility		Low Volatility	
		T_R	T_{SQ}	T_R	T_{SQ}
USDAUD	LSTM	0.8088	0.7436	1.0000	1.0000
	LSTM+Green Factor	0.8088	0.8180	0.0482	0.0120
	LSTM+Brown Factor	1.0000	1.0000	0.0482	0.0140
	LSTM+OVX	0.1380	0.3940	0.7652	0.7656
	LSTM+Green+Brown Factors	0.8088	0.8180	0.0482	0.0120
USDEUR	LSTM	0.7192	0.7792	0.1522	0.2156
	LSTM+Green Factor	0.7020	0.7792	0.1522	0.2156
	LSTM+Brown Factor	1.0000	1.0000	0.1522	0.2156
	LSTM+OVX	0.1284	0.2838	1.0000	1.0000
	LSTM+Green+Brown Factors	0.1284	0.1450	0.0610	0.0748
USDCAD	LSTM	0.7380	0.6492	0.0782	0.0970
	LSTM+Green Factor	1.0000	1.0000	0.0782	0.0970
	LSTM+Brown Factor	0.7380	0.6602	0.7952	0.7990
	LSTM+OVX	0.1626	0.2126	1.0000	1.0000
	LSTM+Green+Brown Factors	0.7380	0.6518	0.0782	0.0774
USDNOK	LSTM	0.6836	0.5692	0.1316	0.1072
	LSTM+Green Factor	0.7616	0.7594	0.0362	0.0092
	LSTM+Brown Factor	1.0000	1.0000	0.0536	0.0364
	LSTM+OVX	0.6836	0.5610	1.0000	1.0000
	LSTM+Green+Brown Factors	0.6836	0.6264	0.1316	0.1072
WTI	LSTM	0.3262	0.3628	0.7124	0.7154
	LSTM+Green Factor	0.3262	0.3628	1.0000	1.0000
	LSTM+Brown Factor	0.3262	0.3628	0.2226	0.2416
	LSTM+OVX	1.0000	1.0000	0.5654	0.4974
	LSTM+Green+Brown Factors	0.3262	0.3628	0.3412	0.3238
BRENT (BCO)	LSTM	0.4690	0.3502	0.5980	0.5898
	LSTM+Green Factor	0.4690	0.3502	0.1852	0.2566
	LSTM+Brown Factor	0.4690	0.3502	1.0000	1.0000
	LSTM+OVX	1.0000	1.0000	0.0312	0.0150
	LSTM+Green+Brown Factors	0.4438	0.2648	0.5980	0.5898

Note: This table reports the MCS (Model Confidence Set) test statistics of Hansen et al. (2011)— T_R and T_{SQ} —for LSTM models across exchange rates and commodity prices, under high and low volatility regimes as determined by a regime switching model. Values are rounded to four decimal places. Higher statistics suggest stronger performance.

Table 15: HAR Regressions with Brown Index and Inventory Interaction

	(1)	(2)	(3)	(4)
Constant	0.0756*** (0.0006)	0.0756*** (0.0006)	0.0756*** (0.0006)	0.0753*** (0.0006)
RV_t^d	0.0335*** (0.0024)	0.0314*** (0.0021)	0.0315*** (0.0021)	0.0314*** (0.0021)
RV_t^w	-0.0499*** (0.0013)	-0.0506*** (0.0013)	-0.0506*** (0.0013)	-0.0507*** (0.0013)
RV_t^m	0.0389*** (0.0019)	0.0390*** (0.0020)	0.0390*** (0.0020)	0.0389*** (0.0020)
$Brown_t$		0.0050*** (0.0015)	0.0051*** (0.0015)	0.0058*** (0.0014)
$Inventory_t$			0.0011 (0.0007)	0.0012* (0.0006)
$Brown_t \times Inventory_t$				-0.0030*** (0.0011)
Adj. R^2	0.5591	0.5651	0.5654	0.5666
Wald test: $H_0: Brown \times Inventory = 0$, $z = -2.70$, $p = 0.007$				

Notes. This table reports results from HAR regressions of next-day realized volatility (RV_{t+1}) on daily (RV_t^d), weekly (RV_t^w), and monthly (RV_t^m) realized volatility components, the Brown Index ($Brown_t$), oil inventories ($Inventory_t$), and their interaction ($Brown_t \times Inventory_t$). Newey–West standard errors (in parentheses) are computed using five lags. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. All variables are standardized.

Table 16: HAR Regressions with Brown Index and Climate Policy Uncertainty (CPU)

	(1)	(2)	(3)	(4)
Constant	0.0846*** (0.0026)	0.0846*** (0.0026)	0.0846*** (0.0026)	0.0843*** (0.0026)
RV_t^w	-0.0423*** (0.0051)	-0.0423*** (0.0051)	-0.0420*** (0.0050)	-0.0422*** (0.0050)
RV_t^m	0.0590*** (0.0043)	0.0590*** (0.0043)	0.0582*** (0.0043)	0.0584*** (0.0042)
Brown _t		0.0119*** (0.0038)	0.0126*** (0.0038)	0.0120*** (0.0034)
CPU _t			-0.0024 (0.0027)	-0.0024 (0.0027)
Brown _t × CPU _t				0.0015 (0.0031)
Adj. R^2	0.5270	0.5291	0.5298	0.5294
Observations	425	425	425	425
Wald test: H_0 : Brown × CPU = 0, $z = 0.49$, $p = 0.625$				

Notes. This table reports HAR regressions of realized volatility (RV_{t+1}) on weekly (RV_t^w) and monthly (RV_t^m) realized volatility components, the Brown Index (Brown_t), the Climate Policy Uncertainty index (CPU_t), and their interaction (Brown_t × CPU_t). Coefficients are reported with Newey–West standard errors (in parentheses). ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. All variables are standardized. The sample covers monthly data from 2015–2023.

Table 17: HAR Regressions with Brown Index and Global Risk (VIX)

	(1)	(2)	(3)	(4)
Constant	0.0756*** (0.0006)	0.0756*** (0.0006)	0.0756*** (0.0006)	0.0752*** (0.0008)
RV_t^d	0.0335*** (0.0024)	0.0314*** (0.0021)	0.0314*** (0.0021)	0.0311*** (0.0019)
RV_t^w	-0.0499*** (0.0014)	-0.0506*** (0.0014)	-0.0506*** (0.0014)	-0.0506*** (0.0014)
RV_t^m	0.0389*** (0.0020)	0.0390*** (0.0021)	0.0391*** (0.0021)	0.0394*** (0.0020)
Brown _t		0.0050*** (0.0015)	0.0056*** (0.0021)	0.0043** (0.0020)
VIX _t			-0.0007 (0.0015)	-0.0006 (0.0015)
Brown _t × VIX _t				0.0004 (0.0007)
Adj. R^2	0.5591	0.5651	0.5651	0.5655
Observations	8,701	8,701	8,701	8,701
Wald test: H_0 : Brown × VIX = 0, $z = 0.60$, $p = 0.55$				

Notes. This table reports results from HAR regressions of next-day realized volatility (RV_{t+1}) on daily (RV_t^d), weekly (RV_t^w), and monthly (RV_t^m) realized volatility components, the Brown Index (Brown_t), the global risk proxy (VIX_t), and their interaction (Brown_t × VIX_t). Coefficients are reported with Newey–West standard errors (in parentheses). ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. All variables are standardized. The sample covers daily data from 2015–2023.